AN ANALYSIS OF A TWO-PERSON INTERACTION SITUATION IN TERMS OF A MARKOV PROCESS

BY

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IN TERMS OF A MARKOV PROCESS [1/]

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The present study represents an attempt to quantitatively describe behavior in a game situation involving <u>social interaction</u> between two individuals. The basis of prediction is in terms of a Markov model for learning which, in conceptual development, is closely related to statistical learning theory (4).

Before proceeding to the details of this study there are three general remarks we would like to make concerning the fundamental ideas and methods. (i) The principles of behavior that constitute our theory of social interaction are rigorously derivable from general principles of individual behavior, in particular, from stochastic versions of reinforcement theory. (ii) The results are quantitative in a sharp sense; elaborate mathematics have not been applied to quantities which can only be ordinally measured. (iii) The underlying principles constitute a genuine theory in the sense that prior to experimentation quantitative predictions of behavior may be made for a wide range of experimental parameter values.

For the purposes of this experiment a play of a game is a trial. On a given trial, each of the players makes a choice between one of two responses. After the players have independently indicated their responses, the outcome of the trial is announced. In an earlier study (2) we considered games where the outcomes were such that on each trial one player was "correct" and the other player was "incorrect"; that is, zero-sum games. In this study the games have outcomes such that on each trial both players can be correct, both can be incorrect, or one can be correct and the other incorrect; that is, non-zero-sum games. More important than the shift from zero-sum to non-zero-sum games is the fact that in contrast to (2) the players were informed that they were interacting with each other.

On all trials, the game is described by the following payoff matrix:

	B_1	B_2
A_1	(x_1,y_1)	(x_2,y_2)
A_2	(x_3,y_3)	(x_4,y_4)

The players are designated A and B. The responses available to player A are A_1 and A_2; similarly, the responses for player B are B_1 and B_2. If player A selects A_1 and player B selects B_1 then there is (i) a probability x_1 that player A is correct and $1-x_1$ that player A is incorrect, and (ii) a probability y_1 that player B is correct and $1-y_1$ that player B is incorrect. The outcome of the other three response pairs A_1B_2, A_2B_1 and A_2B_2 are identically specified in terms of (x_2,y_2), (x_3,y_3) and (x_4,y_4). For subsequent derivations it will be useful to introduce the following notation:

$$
\begin{aligned}
a_i &= x_i y_i \\
b_i &= x_i(1-y_i) \\
c_i &= (1-x_i)y_i \\
d_i &= (1-x_i)(1-y_i)
\end{aligned}
\tag{1}
$$

where $a_i+b_i+c_i+d_i = 1$.

The experiment to be presented employs the procedure outlined above. At the start of an experimental session Ss were informed of the game characteristics of the situation and instructed to maximize the number of trials on which their response was correct.

Our theoretical analysis of behavior in the situation is in terms of a Markov model. Since a detailed mathematical development of the model is presented elsewhere (11) we shall confine ourselves to the most salient features and omit mathematical proofs. A more complete development of the psychological concepts which lead to the present model and its relation to the Estes and Burke stimulus sampling theory can be found in (2).

Model.-- In our situation, where two responses are available to each S, we say that if a response occurs and is correct, then the response is reinforced; if a response occurs and is incorrect then the alternative response is reinforced.

On any trial, a player is described as being in one of two states. If in state 1, he will make response 1; if in state 2, response 2. Thus the two players can be specified in terms of the following four states: $< A_1,B_1 >$, $< A_1,B_2 >$, $< A_2,B_1 >$ and $< A_2,B_2 >$ where the first member of a couple indicates the response of player A and the second, the response of player B.

The learning process is defined with respect to these states.

For player A we assume that when one of his responses is reinforced on trial n there is (i) a probability θ_A that the organism is affected by the reinforcing event so that on trial $n+1$ he will make the response reinforced on trial n and (ii) a probability $1-\theta_A$ that the organism is not affected by the reinforcing event and therefore repeats on trial $n+1$ the response made on trial n. Stated equivalently there is a probability θ_A that the state of the player at the start of trial $n+1$ is the one corresponding to the response reinforced on trial n and $1-\theta_A$ that the state of the player at the start of trial $n+1$ is the same as at the start of trial n regardless of the reinforcing event.

Identical rules describe the learning process for player B in terms of θ_B. Thus, the parameters θ_A and θ_B describe the learning rate characteristics of players A and B respectively, and are quantities which can be estimated from a subset of the data and used to predict the remaining data (6) or in some cases estimated from other experimental set-ups (5).

For the above set of assumptions and the payoff probabilities a_i, b_i, c_i and d_i, the transition matrix (7) describing the learning process can be derived and is as follows:

	$\langle A_1,B_1 \rangle$	$\langle A_1,B_2 \rangle$	$\langle A_2,B_1 \rangle$	$\langle A_2,B_2 \rangle$
$\langle A_1,B_1 \rangle$	$a_1+b_1(1-\theta_B)$ $+c_1(1-\theta_A)$ $+d_1(1-\theta_A)(1-\theta_B)$	$b_1\theta_B$ $+d_1\theta_B(1-\theta_A)$	$c_1\theta_A$ $+d_1\theta_A(1-\theta_B)$	$d_1\theta_A\theta_B$
$\langle A_1,B_2 \rangle$	$b_2\theta_B$ $+d_2\theta_B(1-\theta_A)$	$a_2+b_2(1-\theta_B)$ $+c_2(1-\theta_A)$ $+d_2(1-\theta_A)(1-\theta_B)$	$d_2\theta_A\theta_B$	$c_2\theta_A$ $+d_2\theta_A(1-\theta_B)$
$\langle A_2,B_1 \rangle$	$c_3\theta_A$ $+d_3\theta_A(1-\theta_B)$	$d_3\theta_A\theta_B$	$a_3+b_3(1-\theta_B)$ $+c_3(1-\theta_A)$ $+d_3(1-\theta_A)(1-\theta_B)$	$b_3\theta_B$ $+d_3\theta_B(1-\theta_A)$
$\langle A_2,B_2 \rangle$	$d_4\theta_A\theta_B$	$c_4\theta_A$ $+d_4\theta_A(1-\theta_B)$	$b_4\theta_B$ $+d_4\theta_B(1-\theta_A)$	$a_4+b_4(1-\theta_B)$ $+c_4(1-\theta_A)$ $+d_4(1-\theta_A)(1-\theta_B)$

Rows designate the state on trial n and columns the state on trial $n+1$. Thus $d_2\theta_A\theta_B$, the entry in row 2, column 3 is the conditional probability of being in state $\langle A_2,B_1 \rangle$ on trial $n+1$ given that the pair of Ss was in state $\langle A_1,B_2 \rangle$ on trial n, for we have:

$$d_2\theta_A\theta_B = a_2\cdot 0 + b_2\cdot 0 + c_2\cdot 0 + d_2[(1-\theta_A)(1-\theta_B)\cdot 0 + \theta_A(1-\theta_B)\cdot 0 + \theta_B(1-\theta_A)\cdot 0 + \theta_A\theta_B\cdot 1].$$

The one stage transition probabilities completely describe response behavior in the situation and from these one can obtain any theoretical quantity desired. In particular we can derive an expression for the asymptotic probability of each of the four states, or equivalently stated, the asymptotic probability of the joint occurrence of responses A_i and B_j $(i,j = 1 \text{ or } 2)$ on a trial (7,8); this probability will be denoted as $p_\infty(<A_i,B_j>)$. In terms of these quantities we obtain the asymptotic probability of an A_1 and a B_1 response. Namely,

$$p_\infty(A_1) = p_\infty(<A_1,B_1>) + p_\infty(<A_1,B_2>) \tag{2}$$

$$p_\infty(B_1) = p_\infty(<A_1,B_1>) + p_\infty(<A_2,B_1>). \tag{3}$$

The general expressions for $p_\infty(A_1)$ and $p_\infty(B_1)$ are too lengthy to reproduce here but special forms of the equations will be used in analyzing data of the present study.

The essential interaction character of our Markov model is made clear by the following observation. The joint probabilities $p_n(<A_i,B_j>)$ are fundamental to the process rather than the individual probabilities $p_n(A_i)$ and $p_n(B_j)$. Given the former the latter can be computed, but not vice versa. In particular the players are not responding independently, that is, in general

$$p_n(<A_i,B_j>) \neq p_n(A_i)p_n(B_j).$$

Method

Experimental parameter values.-- Two groups were run; for game-theoretic considerations to be indicated later, they were designated Sure and Mixed. For the Sure Group $x_1 = 1$, $x_2 = \frac{3}{4}$, $x_3 = \frac{1}{4}$, $x_4 = \frac{1}{2}$ and $y_1 = \frac{1}{4}$, $y_2 = 1$, $y_3 = \frac{5}{8}$, $y_4 = \frac{5}{8}$. For the Mixed Group $x_1 = 1$, $x_2 = \frac{3}{8}$, $x_3 = 0$, $x_4 = \frac{5}{8}$ and $y_1 = 0$, $y_2 = 1$, $y_3 = \frac{5}{8}$, $y_4 = \frac{3}{8}$.

Apparatus.-- The apparatus has already been described in detail (2), and only the salient features will be repeated here. The Ss, run in pairs, sat at opposite ends of a table. Mounted vertically in front of each S was a large opaque panel, E sat between the two panels and was not visible to either S. The apparatus, as viewed from the S's side, consisted of two silent operating keys mounted at the base of the panel; upon the panel were mounted three milk glass panel lights. One of these lights, which served as the signal for S to respond, was centered between the keys at S's eye level. Each of the two remaining lights, the reinforcing signals, was mounted directly above one of the keys. The presentation and duration of the lights were automatically controlled.

Subjects.-- The Ss were 88 undergraduates obtained from introductory psychology courses at Stanford University. They were randomly assigned to the experimental groups with the restriction that there were 24 pairs of Ss in the Sure Group and 20 in the Mixed Group.

Procedure.-- For each pair of Ss, one person was randomly selected as player A and the other player B. Further, for each S one of the two response keys was randomly designated response 1 and the other response 2 with the restriction that the following possible combinations occurred equally often in each of the experimental groups: (a) A_1 and B_1 on the right, (b) A_1 on the right and B_1 on the left, (c) A_1 on the left and B_1 on the right, and (d) A_1 and B_1 on the left.

When the Ss had been seated they were read the following instructions:

"This experiment is analogous to a real life situation where what you gain or lose depends not only on what you do but also on what someone else does. In fact, you should think of the situation as a game involving you and another player, the person at the other end of the table.

"The experiment for each of you consists of a series of trials. The top center lamp on your panel will light for about two seconds to indicate the start of each trial. Shortly thereafter one or the other of the two lower lamps will light up. Your job is to predict on each trial which one of the two lower lamps will light and indicate your prediction by pressing the proper key. That is, if you expect the left lamp to light press the left key, if you expect the right lamp to light press the right key. On each trial press one or the other of the two keys but never both. If you are not sure which key to press then guess.

"Be sure to indicate your choice by pressing the proper key immediately after the onset of the signal light. That is, when the signal light goes on press one or the other key down and release it. Then wait until one of the lower lights goes on. If the light above the key you pressed goes on your prediction was correct, if the light above the key opposite from the one you pressed goes on you were incorrect.

"Being correct or incorrect on a given trial depends on the key you press and also on the key the other player presses. With some combinations of your key choice with the other player's key choice, you may both be correct; with other combinations one player will be correct and the other incorrect; for still other combinations you may both be incorrect.

"As you have probably already guessed, the situation is fairly complicated. The object of the experiment is to see how many correct predictions you can get over a series of trials."

Questions were answered by paraphrasing the appropriate part of the instructions.

Following the instructions, 210 trials were run in continuous sequence. For each pair of Ss, sequences of reinforcing lights were generated in accordance with assigned values of (x_i, y_i) and observed responses.

On all trials the signal light was lighted for 3.5 sec; the time between successive signal exposures was 10 sec. The reinforcing light followed the cessation of the signal light by 1.5 sec. and remained on for 2 sec.

Results and Discussion

Mean learning curves and asymptotic results.-- Figure 1 presents the mean proportions of A_1 and B_1 responses in successive blocks of 30 trials for the entire sequence of 210 trials. An inspection of this

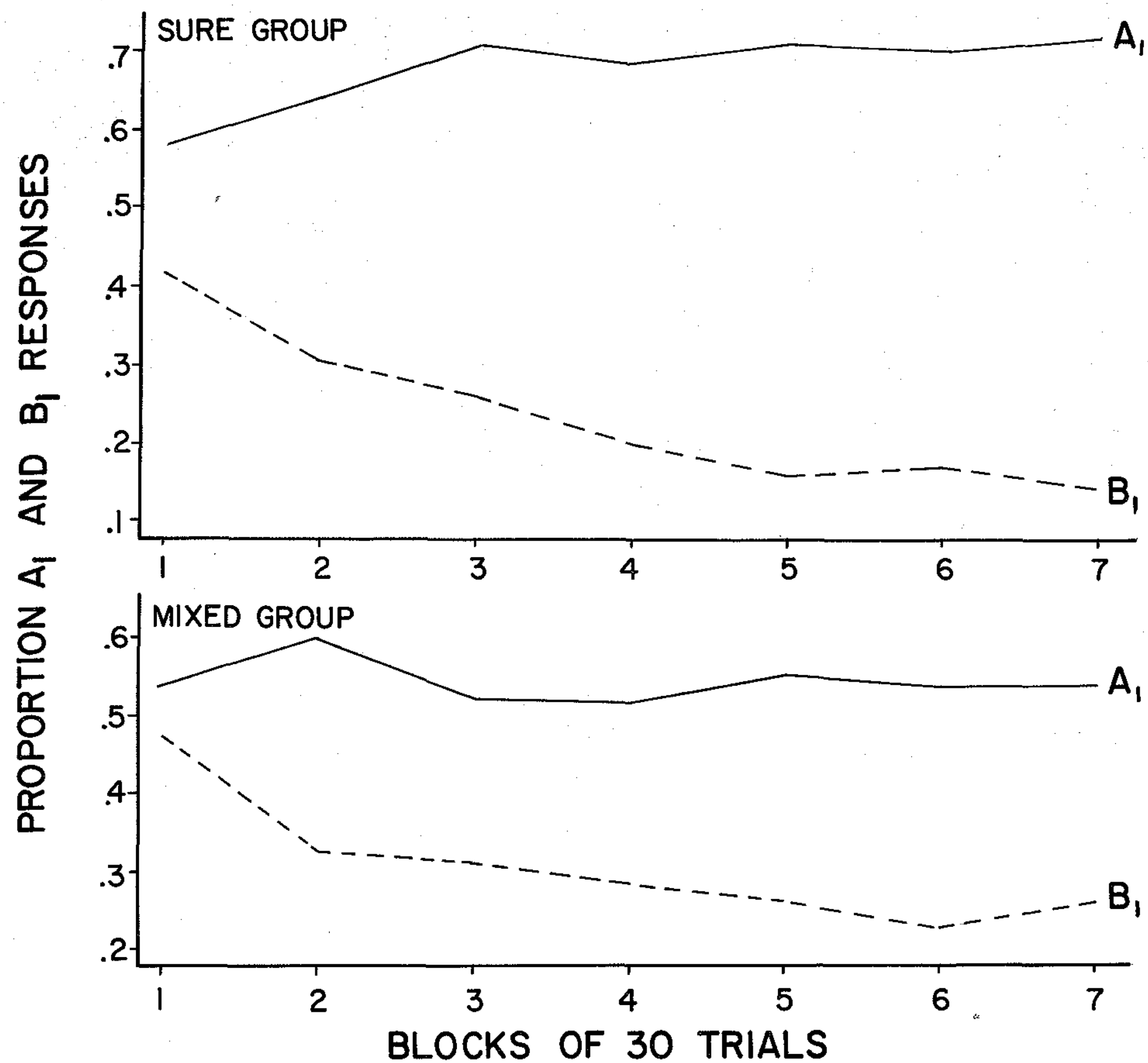
SURE GROUP
MIXED GROUP
A_1
B_1
A_1
B_1
PROPORTION A_1 AND B_1 RESPONSES
.7
.6
.5
.4
.3
.2
.1
.6
.5
.4
.3
.2
1
2
3
4
5
6
7
1
2
3
4
5
6
7
BLOCKS OF 30 TRIALS

figure indicates that responses were fairly stable over the last 90 trials. To check the stability of response behavior for individual data, *t*s for paired measures were computed between response proportions for the first and last halves of the final block of 90 trials. In all cases the obtained values of *t* did not approach significance at the .10 level. In view of these results it appears reasonable to assume that a constant level of responding had been attained; consequently the proportions computed over the last 90 trials were used as estimates of the asymptotic probabilities of an A_1 and a B_1 response. Table 1 presents the observed mean proportions of A_1 and B_1 responses in the last 90 trial block and the standard deviations associated with these means. Entries for the Sure Group are based on N=24; for the Mixed Group, N=20.

The values predicted by the Markov model are also presented in Table 1 and are obtained by substitution in the following equations:

$$p_{\infty}(A_1) = \frac{(1-x_4)(1-y_3) + (1-x_3)(1-y_4)}{(2-x_2-x_4)(2-y_3-y_4) - (x_4-x_3)(y_4-y_2)} \tag{4}$$

$$p_{\infty}(B_1) = \frac{(1-x_2)(1-y_4) + (1-x_4)(1-y_2)}{(2-x_2-x_4)(2-y_3-y_4) - (x_4-x_3)(y_4-y_2)} \tag{5}$$

It should be noted that these equations are not a general solution to the Markov process described in the first section of this paper but represent a solution only when the following pair of conditions are satisfied:[2]

Table 1. Predicted and observed mean proportions of A_1 and B_1 responses over the last block of 90 trials.

	A_1			B_1		
	Predicted	Observed	s	Predicted	Observed	s
Sure	.714	.719	.0904	.143	.158	.0990
Mixed	.551	.546	.0557	.281	.250	.0787

$$x_1+x_3 = x_2 + x_4$$
$$(6) \qquad y_1+y_2 = y_3 + y_4.$$

An inspection of equations (4) and (5) indicates that $p_{\infty}(A_1)$ and $p_{\infty}(B_1)$ are independent of the learning parameters θ_A and θ_B and strictly functions of x_i and y_i. Consequently these equations should account for both individual asymptotic behavior and group mean asymptotic values.

Inspection of Table 1 indicates very close agreement between observed and predicted values. To check this agreement *t* tests were run between the observed and predicted values employing the observed standard deviation of the mean as the error term. In all cases the obtained value of *t* did not approach significance at the .10 level.

A check on the correspondence between individual asymptotic behavior and predicted values is equivalent to evaluating the agreement between observed standard deviations presented in Table 1 and asymptotic variability predicted by the model. Unfortunately direct computation of this theoretical quantity is extremely cumbrous, and we have not obtained an analytical solution. Nevertheless, some results from Monte Carlo runs (2,3) tentatively suggest that the observed variances are of the proper order to be accounted for in terms of the present model.

Transition probabilities.-- Because of the relatively simple mathematical character of stationary Markov processes with a finite number of states, it is possible to ask certain detailed questions. Probably the most immediate question is: how do the aggregate transition matrices for the two experimental

Table 2. Observed transition matrices corresponding to the theoretical transition matrix. Computed over the last 90 trials.

	Sure				Mixed			
	$\langle A_1,B_1\rangle$	$\langle A_1,B_2\rangle$	$\langle A_2,B_1\rangle$	$\langle A_2,B_2\rangle$	$\langle A_1,B_1\rangle$	$\langle A_1,B_2\rangle$	$\langle A_2,B_1\rangle$	$\langle A_2,B_2\rangle$
$\langle A_1,B_1\rangle$	.24	.59	.06*	.11*	.19	.62	.06*	.13*
$\langle A_1,B_2\rangle$	.07*	.72	.01*	.20	.05*	.45	.03*	.47
$\langle A_2,B_1\rangle$	.20	.38	.17	.25	.26	.34	.20	.20
$\langle A_2,B_2\rangle$	.16	.42	.08	.34	.16	.39	.12	.33

groups compare with the theoretical matrix derived in the first section of this paper. Table 2 presents the observed matrices computed over the last 90 trials for the two groups. No statistical test is needed to see that the observed matrices differ significantly from the theoretical matrix. It suffices to observe that in the theoretical matrix, for the set of experimental parameter values employed in both the Mixed and Sure Groups, the transition probabilities in the last two entries in row one and the first and third entries in row two are identically zero, but in the observed matrices, entries in these cells (denoted by * in Table 2) are in some cases markedly different from zero.

Without regard to a specific model we can ask another highly relevant question about the data: can the data be more adequately accounted for by a two-stage Markov model which employs information about responses of *Ss* on the previous two trials as compared with a one-stage model which employs response information about only one preceding trial? For this purpose we use the test described in (*1*). The null hypothesis is that $p_{ijk} = p_{jk}$ for $i=1,2,3,4$ where p_{ijk} is the probability of state k given, successively, i and j on the two previous trials, and p_{jk} is the probability of state k simply given state j on the preceding trial. To test this hypothesis the following sum was computed for the aggregate group data:

$$\chi^2 = \sum_{i,j,k} n^*_{ij}(\hat{p}_{ijk} - \hat{p}_{jk})^2/\hat{p}_{jk} ,$$

where $n^*_{ij} = \sum_k n_{ijk}$. If the null hypothesis is true, χ^2 has the usual limiting distribution with $4(4-1)^2 = 36$ degrees of freedom.

The values of χ^2 were 51.9 for the Sure Group and 49.7 for the Mixed Group. In neither case were these values significant at the .05 level. This result indicates that for the present set of data there is no statistically significant improvement in prediction if one knows the response history of the pair of Ss on the previous two trials as compared to such knowledge on only one preceding trial. The Markov model presented in this paper is formulated as a one-stage process, but it should be pointed out that this assumption is not necessary for our general theoretical approach to the problem of interaction among Ss in such situations.

Game theory comparisons.-- The development of an adequate theory of optimal strategies for non-zero-sum, two-person games has been intensively pursued in the past decade, but as yet no concept of optimality has been proposed which is as solidly based as the minimax concept for zero-sum, two-person games. A natural division of non-zero-sum games is into cooperative and non-cooperative games. In a cooperative game the players are permitted to communicate and bargain before selecting a strategy; in a non-cooperative game no such communication and bargaining is permitted. As should be obvious from earlier sections, the experimental situation described in this paper corresponds to a non-cooperative rather than a cooperative game.

In certain special non-zero-sum games the highly appealing sure-thing principle may be used to select an optimal strategy. In brief, a strategy satisfies the sure-thing principle if no matter what your opponent does you are at least as well off, and possibly better off, with this strategy in comparison to any other available to you. The experimental parameters (x_i, y_i) were so selected that for one of the experimental groups, namely the Sure Group, each S had available such a strategy, namely, response A_1 for player A and B_2 for player B with probability 1.

Unfortunately in most non-zero-sum games the sure-thing principle does not lead to selection of a unique optimal strategy, or even to a relatively small class of optimal strategies. In this event, probably the best concept of optimality yet proposed for non-cooperative, non-zero-sum games is Nash's notion of an equilibrium point (9,10). Roughly speaking, an equilibrium point is a set of strategies, one for each player, with the property that these strategies provide a way of playing the game such that if all the players but one follow their given strategies, the remaining player cannot do better by following any strategy other than one belonging to the equilibrium point. The experimental parameters (x_i, y_i) were selected for the second experimental group, the Mixed Group, so that the game had a unique equilibrium point consisting of a mixed strategy for each S. In particular, player A should have chosen response A_1 and player B, response B_1, with probability $1/5$.

Although Ss were not shown the pay-off matrix in our experiment, it is a reasonable conjecture that after a large number of trials they would learn enough about the situation to approach an optimal game strategy, namely, a sure-thing strategy for one group, and an equilibrium point for the other. Concerning this conjecture the results for the Sure Group seem conclusive: the optimal strategies of responding A_1 or B_2 with probability 1, for players A or B respectively, are not even roughly approximated by the observed asymptotic means. Results for the Mixed Group are also decisive. The observed mean asymptotic probability of an A_1 response differs significantly from the equilibrium point strategy of 1/5 at beyond the .001 level. And the observed mean asymptotic probability of a B_1 response differs significantly from the equilibrium point strategy at the .02 level.

Comments.-- From the standpoint of many social psychologists the experimental situation used in this study is too highly structured in terms of successful performance, and interaction between Ss is too severely restricted. Concepts like those of friendliness, cohesiveness, group pressure, opinion discrepancy and receptivity, which have been important in numerous recent investigations, play no role in our situation. However, these limitations are offset by some substantial assets. An intrinsically quantitative prediction of behavior in an interaction situation has been derived in a rigorous manner from fundamental principles of reinforcement and associative learning. In particular the only psychological concepts needed for the analysis of our experiment are the classical triad of stimulus, response and reinforcement. In comparison, studies using common-sense group concepts like those just mentioned have not been quantitative in character nor have they made any serious headway toward deriving these concepts from any specific psychological theory.

From another viewpoint it is interesting to observe that this study supports results in (2), namely, that various concepts of optimal strategy from the theory of games have not proved useful tools for the prediction of actual behavior. Although this generalization must be qualified by the remarks that Ss were not shown the pay-off matrix of the game, the relative success of statistical learning models in predicting behavior seems substantial. Still, it is of theoretical interest to find out how much, if any, explicit knowledge of the pay-off matrix disturbs the predictive accuracy of the learning model; experimental investigation of this problem is now under way.

Summary

The study deals with an analysis of a non-zero-sum, two-person game situation in terms of a Markov model for learning. The formulation of this model is derived from considerations similar to those employed by Estes and Burke in their stimulus sampling approach to learning.

Ss were run in pairs for 210 trials. A single play of the game was treated as a trial. On a trial, each player made a choice between one of two alternative responses; after the players had made their response, the outcome of the trial was announced. The responses available to player A were designated A_1 and A_2; similarly the responses available to player B were B_1 and B_2. If player A selected A_1 and player B selected B_1, then (i) there was a probability x_1 that player A was correct and $1-x_1$ that player A was incorrect, and (ii) there was a probability y_1 that player B was correct and $1-y_1$ that player B was incorrect. The outcome of the other three response pairs A_1B_2, A_2B_1, and A_2B_2 were identically specified in terms of (x_2,y_2), (x_3,y_3), and (x_4,y_4). Ss were informed of the game characteristics of the situation and instructed to maximize the number of trials on which their response was correct.

Two groups were run, each employing a different set of (x_i,y_i) values. The selection of these values was determined by game-theoretic considerations; that is, a group had available either a mixed equilibrium point strategy or a sure-thing strategy.

Analysis of the results was in terms of the following comparisons between theory and data: (a) mean asymptotic response probabilities, Summary (Cont.)

(b) one and two stage transition probabilities, and (c) variances associated with asymptotic response probabilities. In general, the predicted and observed results were in close agreement. The results of the study were also considered from the viewpoint of game theory.

Footnotes

1/ This research was supported by the Behavioral Sciences Division of the Ford Foundation and by the Group Psychology Branch of the Office of Naval Research.

2/ In general, the solutions for $p_{\infty}(A_1)$ and $p_{\infty}(B_1)$ are functions of both θ_A and θ_B (2,11).

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